Greek Letters

β = empirical constant used in Eq. 13
λ = lateral microscale of turbulence
ν = kinematic viscosity
ξ = x/M, dimensionless distance from grid
ψ = $r/\sqrt{2}\lambda$, dimensionless distance between two points

Subscripts and Superscripts

0 = initial value
, = partial differentiation; $u_{i,j} = \partial u_i/\partial x_j$, $u_{i,jj} = \partial^2 u_i/\partial x_j^2$, *etc.*
i,j,k = Cartesian coordinate components
$-$ = time-smoothed value
= root-mean-square value

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Heat Transfer to a Laminar Flow Fluid in a Circular Tube

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The Graetz (1883, 1885) problem involved the finding of the temperature profile in a fully-developed laminar flow of fluid inside a circular tube. In this communication, we present a general analytical solution in closed form via the method of variable transformation. Also theoretical expressions of Nusselt number (arithmetic mean and logarithmic mean) as a function of Graetz number were obtained.

GRAETZ PROBLEM

The governing equation for the Graetz problem may be obtained from an energy balance in cylindrical coordinates. For a fluid with constant physical properties, neglecting axial conduction, and at steady state, the resulting partial differential equation in the dimensionless form is:

$$(1-\xi^2)\frac{\partial\theta}{\partial\zeta} = \frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\xi\frac{\partial\theta}{\partial\xi}\right) \tag{1}$$

with boundary conditions:
1. at $\xi = 0$, θ is finite,
2. at $\xi = 1$, $\theta = 0$,
3. at $\xi = 0$, $\theta = 1$,

with

$$\theta = \frac{T_w - T}{T_w - T_o}, \quad \xi = \frac{r}{r_1} \quad \text{and} \quad \zeta = \frac{kz}{\rho c_p v_{\max} r_1^2}$$

By the method of separation of variables, we let

$$\theta = Z(\zeta)R(\xi) \tag{2}$$

Equation 1 may be decomposed to the following two ordinary differential equations,

$$\frac{dZ}{Z} = -\beta^2\zeta \tag{3}$$

$$\xi\frac{d^2R}{d\xi^2} + \frac{dR}{d\xi} + \beta^2\xi(1-\xi^2)R = 0 \tag{4}$$

where β^2 is a positive, real number and constitutes an eigenvalue of the system.

The solution of Eq. 3 is

$$Z = c_1 e^{-\beta^2\zeta} \tag{5}$$

where c_1 is an arbitrary constant.

To solve Eq. 4, the following transformations of both dependent and independent variables are performed:

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(I) Let $v = -\beta\xi^2$,
(II) Let $R(v) = e^{-v/2}S(v)$,
This transforms Eq. 4 into

$$v\frac{d^2S}{dv^2} + (1 - v)\frac{dS}{dv} - \left(\frac{1}{2} + \frac{\beta}{4}\right)S = 0 \qquad (6)$$

Equation 6 is in the form of the confluent hypergeometric equation known as Kummer's equation (Slater, 1960).

The two linearly independent solutions of Eq. 6 are transformed inversely to the original variables. The first boundary condition eliminates one of the independent solutions. The solution of Eq. 4 becomes

$$R = c_2 e^{-\beta\xi^2/2}\, {}_1F_1\left(\frac{1}{2} - \frac{\beta}{4}; 1; \beta\xi^2\right) \qquad (7)$$

$${}_1F_1\left(\frac{1}{2} - \frac{\beta_n}{4}; 1; \beta n\right) = 0 \qquad (8)$$

with $n = 1, 2, 3, \ldots$

The eigenvalues, β_n, are roots of Eq. 8.

Since the system is linear, a general solution of Eq. 2 may be obtained by superposition as follows:

$$\theta = \sum_{n=1}^{\infty} C_n\, e^{-\beta^2\zeta} \cdot e^{-\beta_n\xi^2/2} \cdot {}_1F_1\left(\frac{1}{2} - \frac{\beta_n}{4}; 1; \beta_n\xi^2\right) \qquad (9)$$

The coefficients of solution, C_n, may be obtained by making use of the orthogonal properties of the Sturm-Liouville system after the initial condition is applied.

$$C_n = \frac{\left(\frac{1}{2} - \frac{1}{\beta_n}\right) e^{-\beta_n/2} \cdot {}_1F_1\left(\frac{3}{2} - \frac{\beta_n}{4}; 2; \beta_n\right)}{\int_0^1 (\xi - \xi^3)e^{-\beta_n\xi^2} \cdot \left[\frac{1}{2} - \frac{\beta_n}{4}; 1; \beta_n\xi^2\right]^2 d\xi} \qquad (10)$$

The integrals in the denominator of Eq. 10 can be evaluated by numerical integration.

APPLICATION

From the temperature profile of Eq. 9, the heat flux at wall, the total rate of heat transfer and the bulk temperature of the fluid at the exit can be evaluated. Based on the arithmetic mean of terminal temperature differences and the definition of heat transfer coefficient, the arithmetic mean Nusselt number, $N_{Nu,a} \equiv haD/k$, can be expressed as a function of the Graetz number,

$$N_{Gr} \equiv \frac{\pi\zeta c_p v_{\max} R^2}{2kL}.$$

$$N_{Nu,a} = \frac{2N_{Gr}\sum_{n=1}^{\infty} C_n e^{-\beta_n/2}\left(1 - \frac{2}{\beta_n}\right) {}_1F_1\left(\frac{3}{2} - \frac{\beta_n}{4}; 2; \beta_n\right) \cdot \left[1 - \exp\left(-\frac{\pi\beta_n^2}{2N_{Gr}}\right)\right]}{\pi\left[\frac{1}{2} + \sum_{n=1}^{\infty} c_n e^{-\beta_n/2}\left(1 - \frac{2}{\beta_n}\right) {}_1F_1\left(\frac{3}{2} - \frac{\beta_n}{4}; 2; \beta_n\right) \cdot \exp\left(-\frac{\pi\beta_n^2}{2N_{Gr}}\right)\right]} \qquad (11)$$

Similarly, the logarithmic Nusselt number, $N_{Nu,ln} = hlnD/k$ is obtained as

$$N_{Nu,ln} = \frac{2N_{Gr}\sum_{n=1}^{\infty} c_n e^{-\beta_n/2} \cdot \left(1 - \frac{2}{\beta_n}\right) {}_1F_1\left(\frac{3}{2} - \frac{\beta_n}{4}; 2; \beta_n\right) \cdot \left[1 - \exp\left(-\frac{\pi\beta_n^2}{2N_{Gr}}\right)\right]}{\pi\left[-1 + \sum_{n=1}^{\infty} 2c_n e^{-\beta_n/2}\left(1 - \frac{2}{\beta_n}\right) {}_1F_1\left(\frac{3}{2} - \frac{\beta_n}{4}; 2; \beta_n\right) \cdot \exp\left(-\frac{\pi\beta_n^2}{2N_{Gr}}\right)\right.}$$
$$\left. \times\, l_n\left[\sum_{n=1}^{\infty} 2\, c_n e^{-\beta_n/2}\left(1 - \frac{2}{\beta_n}\right) {}_1F_1\left(\frac{3}{2} - \frac{\beta_n}{4}; 2; \beta_n\right) \cdot \exp\left(-\frac{\pi\beta_n^2}{2N_{Gr}}\right)\right]\right] \qquad (12)$$

Our theoretically predicted relationships between the Nusselt number versus the Graetz number were plotted in Figure 1 from Eqs. 11 and 12 with Graetz number ranging from 1 to 10,000. Experimental] data (Seider and Tate, 1936) were presented on Figure 1 which shows excellent agreement between our theoretical prediction and experimental results. Empirical correlation (Colburn, 1933) to the two dimensionless groups is also presented to show its applicability for Graetz numbers larger than 10. Finally, the applicable regions of two asymptotic approximate solutions of theoretical prediction for long tubes (Graetz, 1883) and for short tubes (Leveque, 1928) are shown in the low and in the high values of Graetz number respectively.

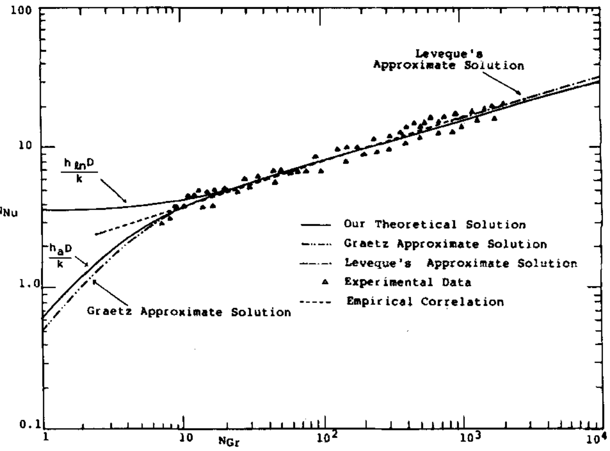

Figure 1. Theoretical heat transfer correlation for Laminar flow in a round tube and its comparison with other methods of prediction.

NOTATION

a	= parameter of confluent hypergeometric function
b	= parameter of confluent hypergeometric function
c_p	= heat capacity
C_n	= coefficient of solution defined in Eq. 10
${}_1F_1(a;b;x)$	= standard confluent hypergeometric function
k	= thermal conductivity
L	= length of the circular tube
r	= radial direction of the cylindrical coordinates
r_1	= radius of the circular tube
T	= temperature of the fluid inside a circular tube
T_o	= temperature of the fluid entering the tube
T_w	= temperature of the fluid at the wall of the tube
$v_{\max}$	= maximum axial velocity of the fluid

Greek Letters

β_n	= dimensionless temperature
ζ	= dimensionless axial direction
θ	= dimensionless temperature
ξ	= dimensionless radial direction
ρ	= density of the fluid
μ	= viscosity of the fluid

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Direct Contact Heat Transfer with Phase Change: Motion of Evaporating Droplets

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Direct contact heat transfer between liquids has the advantage of eliminating metallic heat transfer surfaces which are prone to corrosion and fouling. In addition, if phase change occurs, a larger heat capacity for heat absorption is available. The mechanism of heat transfer between two immiscible phases and dynamics of a vaporizing drop are, relatively, more complex than either that of a drop or a bubble of constant radius.

As reported by Sideman and Taital (1964), Simpson et al. (1974), and Sambi (1981), a dispersed liquid droplet changes its shape from spherical through ellipsoidal to a cap-shaped bubble during the course of evaporation through continuous immiscible liquid medium. In addition to these changes in shape, the two-phase bubble oscillates causing the unevaporated dispersed liquid in the two-phase bubble to slosh from side to side. The combined effect of irregular transformation in the shape of the two-phase bubble, sloshing of the unevaporated liquid in the two-phase bubble, and its zigzag trajectory has complicated the study of the mechanism of heat transfer and dynamics of a vaporizing drop.

As far as theoretical expressions for heat transfer involved, motion of the evaporating two-phase bubble, and the total time of its evaporation are concerned, the complicated nature of the problem has stood in the way of workers in developing suitable models that could represent the experimental data.

In pursuit of such a study, a mathematical model for the heat transfer coefficient has already been developed (Raina and Grover, 1982).

This paper deals with the motion of the vaporizing two-phase bubble.

Based on the results of their experimental studies of the motion of expanding bubbles, Sideman and Taitel (1964) developed the following empirical formula for the pentane-water system describing the relationship between the position of the bubble and time.

$$H = H_0 + Bt^p$$

or

$$U = \frac{dH}{dt} = Bpt^p \tag{1}$$

Quantitative description of the behavior of single bubble and drop dispersions in continuous fluid is often based on the description of a free motion of a single bubble or drop whose shape is usually defined by D of the volume equivalent sphere, that is

$$D = \left(\frac{6V}{\pi}\right)^{1/3} \tag{2}$$

The basic equation for the terminal velocity of a single drop is derived from the force balance equation

$$\pi D^3(\rho_c - \rho_d)\frac{g}{6} = C_D\left(\frac{\pi D^2}{4}\right)\left(\rho_c \frac{U^2}{2}\right)$$

or

$$U = \left[\frac{4}{3}\left(\frac{\rho_c - \rho_d}{\rho_d}\right)\frac{D\cdot g}{C_D}\right]^{1/2} \tag{3}$$

For an evaporating drop in another immiscible liquid, the expression can be written as

$$U = \left[\frac{4}{3}\left(\frac{\rho_c - \rho_{d\text{av}}}{\rho_d}\right)\frac{D\cdot g}{C_D}\right]^{1/2} \tag{4}$$

Investigation of energy equations using the experimental data of Sideman and Taitel (1964) and Sambi (1981) revealed that the heat transfer coefficient is higher for drops of small initial diameter, Figure 1. It was thought worthwhile to put the above expression in a form that would include initial diameter of the drop.

$$\frac{\rho_c}{\rho_{d\text{av}}} = \frac{\rho_c}{\left[\frac{\pi}{6}D_0^3\rho_d \Big/ \frac{\pi}{6}D^3\right]} = \frac{\rho_c}{\rho_d}\left(\frac{D}{D_0}\right)^3$$

or

$$\left(\frac{\rho_c - \rho_{d\text{av}}}{\rho_c}\right) = 1 - \frac{\rho_{d\text{av}}}{\rho_c} = 1 - \frac{\rho_d}{\rho_c}\left(\frac{D_0}{D}\right)^3 \tag{5}$$

Substituting Eq. 5 in Eq. 4 we obtain

$$U = \left[\frac{4}{3}\left\{1 - \frac{\rho_d}{\rho_c}\left(\frac{D_0}{D}\right)^3\right\}\left\{\frac{D\cdot g}{C_D}\right\}\right]^{1/2} \tag{6}$$

For bubbles ($R \geqslant 0.7$ mm) whose shape is nearly spherical, it is most convenient to determine the coefficient of resistance from the relation obtained by Levich (1949, 1952) that is

$$C_D = \frac{2}{3}\left(\frac{\rho_c g D^2}{1.82\sigma_c}\right) \tag{7}$$

where σ_c is the surface tension at the bubble surrounding liquid boundary. Selecki and Gradon (1976) have used this relation in their theoretical expression for the motion of the two-phase bubble.

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